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FREE VIBRATION ANALYSIS OF BENDING STIFF COMPOSITE CONICAL SHELLS WITH DELAMINATION – A FINITE ELEMENT APPROACH

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ABSTRACT

Finite element method is employed for free vibration analysis of bending stiff composite conical shells with delamination using Mindlin's theory. A generalized dynamic equilibrium equation is derived from Lagrange's equation of motion neglecting Coriolis Effect for moderate rotational speeds. An eight noded isoparametric plate bending element is considered for the analyses. The multipoint constraint algorithm is employed to ensure the compatibility of deformation and equilibrium of resultant forces and moments at the delamination crack front. The QR iteration algorithm is utilized for solution of standard eigen value problem. Finite element codes are developed to obtain the non-dimensional natural frequencies of delaminated composite conical shallow shells. The mode shapes for a typical laminate configuration are also depicted.

Keywords: Delamination, Finite Element, Conical Shell, Bending Stiff

1. INTRODUCTION

Composite materials have been increasingly used over the past few decades in a variety of structures that requires specific stiffness and strength. By bonding together with various fibre orientation, directionally dependent properties of composites can be tailored which leads to additional advantages of composites over the use of conventional engineering materials. In the midst of several advantages of composite materials, they are also prone to a wide range of defects and damage which mayt significantly reduce their structural integrity. Delamination is the most common feared damage mode which is unavoidably incurred in practical situation for composite structures. The delaminated composite laminated structures exhibit new vibration characteristics. The presence of invisible delamination can be detected with the help of prior knowledge of natural frequencies and the size and location of delamination can be estimated. In order to ensure operational safety, a profound understanding of dynamic characteristics of composite conical shells is essential for designers.

Twisted cantilever composite shallow conical shells can be idealized as turbo-machinery blades. Rotating pretwisted conical shells with low aspect ratio can be idealized as turbo-machinery blades. The first established work on pretwisted composite plates was carried out by Qatu and Leissa [11] to determine the natural frequencies of stationary plates using laminated shallow shell theory using Ritz method. Liew et al. [7] investigated on pretwisted conical shell to find out the vibratory characteristics of stationary conical shell by using Ritz procedure and by using the same method; the first known three dimensional continuum vibration analysis including full geometric non-linearities and centrifugal accelerations in composite blades was carried out by McGee and chu [8]. Regarding delamination model, two worth mentioning investigations were carried out. It included analytical and experimental determination of natural frequencies of delaminated composite beam by Shen and Grady [12] and the second one dealt with finite element treatment of the delaminated composite cantilever beam and plate by Krawczuk et al. [5] for free vibration analyses. There is not only significant work incurred on single delamination, but also on multiple delaminations. Considering multiple delaminations, failure analysis of composite plate due to bending and impact was numerically investigated by Parhi et al. [9] using finite element method.

A limited investigation have been carried out related to laminated composite cantilever conical shells with initial twist and subsequently the research findings are very limited and scanty. Hence the present endeavour is to work on delaminated cantilever conical shell with different extents of delaminations The present analysis aimed at a parametric study on the free vibration behavior of graphite-epoxy bending stiff composite twisted shallow conical shells having delamination without taking care of the effect of dynamic contact. The undelaminated region is modeled by a single layer of plate elements while the delaminated region is modeled using two layers of plate elements whose interface contains the delamination.

2. THEORETICAL FORMULATION

A shallow shell is characterized by its middle surface which is defined by the equation (6),

$$z = -\frac{1}{2}\left[\frac{x^2}{R_x} + 2\frac{xy}{R_{xy}} + \frac{y^2}{R_y}\right] \qquad (1)$$

The radius of twist (R_{xy}), length (L) of shell and twist angle (Ψ) are related as,

$$\tan\psi = -\frac{L}{R_{xy}} \qquad (2)$$

The dynamic equilibrium equation for moderate rotational speeds neglecting Coriolis effect is derived employing Lagrange's equation of motion and the equation in global form is expressed as [4]

$$[M]\{\ddot{\delta}\} + ([K] + [K_\sigma])\{\delta\} = \{F(\Omega^2)\} \qquad (3)$$

$\{F(\Omega^2)\}$ is the nodal equivalent centrifugal forces. $[K_\sigma]$ depends on the initial stress distribution and is obtained by the iterative procedure upon solving,

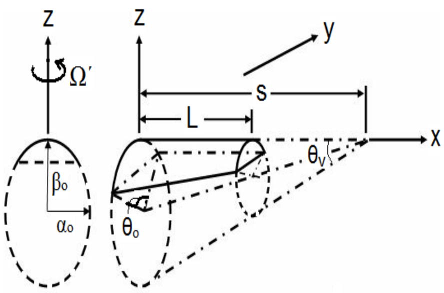
$$([K] + [K_\sigma])\{\delta\} = \{F(\Omega^2)\} \qquad (4)$$

The natural frequencies are determined from the standard eigenvalue problem [1] which is represented below and is solved by QR iteration algorithm,

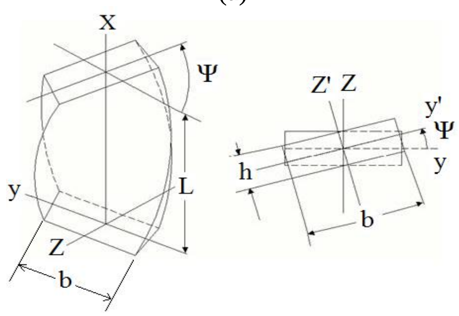
$$[A]\{\delta\} = \lambda\{\delta\} \qquad (5)$$

$$\text{where, } [A] = ([K] + [K\sigma])^{-1}[M]$$

$$\lambda = 1/\omega_n^2 \qquad (6)$$



(a)



(b)

Fig 1.(a) Untwisted conical shell and (b) Twisted plate

3. MULTI-POINT CONSTRAINTS

The plate elements at a delamination crack front is shown in fig. 2. The nodal displacements of elements 2 and 3 at the crack tip are expressed as [2]

$$u_j = u'_j - (z - z'_j)\,\theta_{xj} \qquad (7)$$

$$v_j = v'_j - (z - z'_j)\,\theta_{yj} \qquad (8)$$

$$w_j = w'_j \qquad (\text{where, } j = 2, 3) \qquad (9)$$

The above equation also holds good for element 1 and z'_1 equal to zero. The transverse displacements and rotations at a common node have values expressed as [2],

$$w_1 = w_2 = w_3 = w \qquad (10)$$

$$\theta_{x1} = \theta_{x2} = \theta_{x3} = \theta_x \qquad (11)$$

$$\theta_{y1} = \theta_{y2} = \theta_{y3} = \theta_y \qquad (12)$$

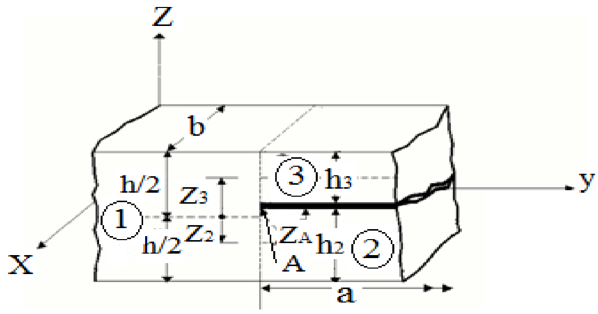


Fig 2. Plate element with delamination crack tip

In-plane displacements of all three elements at crack tip are equal and they are related as [2]

$$u'_2 = u'_1 - z'_2\,\theta_x \qquad (13)$$

$$v'_2 = v'_1 - z'_2\,\theta_y \qquad (14)$$

$$u'_3 = u'_1 - z'_3\,\theta_x \qquad (15)$$

$$v'_3 = v'_1 - z'_3\,\theta_y \qquad (16)$$

where u'_1 is the mid-plane displacement of element 1. Equations of (10) to (16) relating nodal displacements and rotations of elements 1, 2 and 3 at the delamination crack tip, are the multipoint constraint equations used in finite element formulation to satisfy the compatibility of displacements and rotations. Mid-plane strains between elements 2 and 3 are related as [2],

$$\{\varepsilon'\}_j = \{\varepsilon'\}_j + z'_j\{k\} \qquad (17)$$

where $\{\varepsilon\}$ represents the strain vector and $\{k\}$ is the curvature vector being identical at the crack tip for elements 1, 2 and 3. This equation can be considered as a special case for element 1 and z'_1 is equal to zero. In-plane stress-resultants, {N} and moment resultants, {M} of elements 2 and 3 can be expressed as [2],

$$\{N\}_j = [A]_j \{\varepsilon'\}_1 + (z'_j [A]_j + [B]_j) \{k\} \quad (18)$$

$$\{M\}_j = [B]_j \{\varepsilon'\}_1 + (z'_j [B]_j + [D]_j)\{k\} \quad (19)$$

The resultant forces and moments at the delamination front for the elements 1, 2 and 3 satisfy the following equilibrium conditions,

$$\{N\} = \{N\}_1 = \{N\}_2 + \{N\}_3 \quad (20)$$

$$\{M\}=\{M\}_1=\{M\}_2 + \{M\}_3 + z'_2 \{N\}_2 + z'_3 \{N\}_3 \quad (21)$$

$$\{Q\} = \{Q\}_1 = \{Q\}_2 + \{Q\}_3 \quad (22)$$

An eight noded isoparametric quadratic plate bending element with five degrees of freedom at each node (three translation and two rotations) is employed wherein the shape functions are as follows [1]

$$N_i = (1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i + \eta\eta_i - 1)/4 \text{ (for i=1 to 4)} \quad (23)$$

$$N_i = (1 - \xi^2)(1+ \eta\eta_i)/2 \quad \text{(for i=5, 6)} \quad (24)$$

$$N_i = (1- \eta^2)(1 + \xi\xi_i)/2 \quad \text{(for i=6, 8)} \quad (25)$$

4. RESULTS AND DISCUSSION

Non-dimensional natural frequencies for conical shells having a curvature ratio (b_o/Ry) of 0.5 and a thickness ratio (s/h) of 1000 are obtained corresponding to different speeds of rotation **Ω**=0.0, 0.5 and 1.0 with relative distance, d/L=0.33, 0.5 and 0.67. Material properties of graphite-epoxy composite [10] considered as E_1=138.0 GPa, E_2=8.96 GPa, ν=0.3, G_{12}=7.1 GPa, G_{13}=7.1 GPa, G_{23}=2.84 GPa.

4.1 Validation

Computer codes are developed based on present finite element method. The numerical results obtained are compared and validated with the results of published literature [7, 11, 3] as furnished in Table 1, Table 2 and Table 3. . Table 1 presents the convergence study for validation of NDFF of graphite-epoxy composite pretwisted shallow conical shells while Table 2 validates NDFF of three layered graphite- epoxy twisted plates. Table 3 represents NDFF of graphite-epoxy bending stiff composite rotating shells with 25% delamination located at several positions.

The comparative study shows an excellent agreement with the published results and hence it demonstrates the capability of the computer codes developed and proves the accuracy of the analyses. The lower mesh size (6 x 6) consisting of 36 elements and 133 nodes, has been used for the analysis due to computational efficiency. The total number of degrees of freedom involved in the computation is 665 as each node of the isoparametric element is having five degrees of freedom comprising of three translations and two rotations. It is observed from the convergence study that uniform mesh divisions of (6 x 6) and (8 x 8) considering the complete planform of the shell provide nearly equal results the difference being around one percent (1%).

Table 1: NDFF [$\omega=\omega_n b_o^2 \sqrt{(\rho h/D)}$, $D=Eh^3/12(1-\nu^2)$] of graphite-epoxy composite pretwisted conical shells with ν=0.3, s/h=1000, θv=15°, θo=30°

ψ	L/s	Present FEM (8x8)	Present FEM (6x6)	Liew et al. [7]
0°	0.6	0.3524	0.3552	0.3599
	0.7	0.2991	0.3013	0.3060
	0.8	0.2715	0.2741	0.2783
30°	0.6	0.2805	0.2834	0.2882
	0.7	0.2507	0.2528	0.2575
	0.8	0.2364	0.2389	0.2417

Table 2: NDFF of three layered [θ, -θ, θ] graphite- epoxy twisted plates, L/b_o=1, b_o/h=20, ψ=30°

θ	Present FEM	Qatu et al. [11]
15°	0.8618	0.8759
30°	0.6790	0.6923
45°	0.4732	0.4831
60°	0.3234	0.3283

Table 3: NDFF [$\omega=\omega_n L^2 \sqrt{(\rho/E_1h^2)}$] of graphite-epoxy bending stiff composite rotating cylindrical shells with 25% delamination located at several positions [0°/0°/30°//-30°/-30°/30°/0°//0°], a/b_o=1, b_o/h=100, b_o/R_y=0.5 (where '//' indicates location of delamination)

Ω	Present FEM	Karmakar et al. [3]
0.0	1.9316	1.9332
0.5	2.1080	2.1921
1.0	2.6735	2.7475

4.2 Effect of Twist and Rotational Speed

The parametric study is conducted considering combined effect of twist and rotational speed for eight layered graphite-epoxy bending stiff (0°/0°/30°/-30°)s composite conical shells with delamination (Table 4).

Table 4: NDFF [$\omega=\omega_n L^2 \sqrt{(\rho/E_1h^2)}$] of graphite-epoxy delaminated bending stiff conical shells for various twist angles and laminate thicknesses, n=8, a/L=0.33, d/L=0.5, L/s=0.7, θ_o=45°, θ_v=20°.

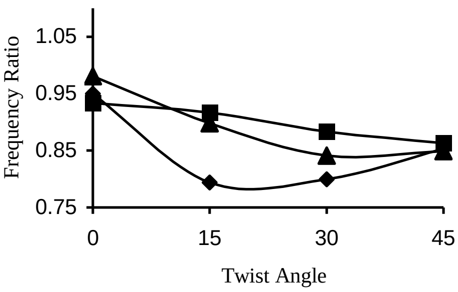
h	ψ	ND	With Delamination		
			Ω=0	Ω=0.5	Ω=1.0
0.0010	0°	0.5192	0.4930	0.0849	0.6769
	15°	0.4172	0.3311	0.3552	0.6937
	30°	0.3007	0.2404	0.0965	0.0584
	45°	0.2166	0.1847	0.3862	0.1248
0.0020	0°	0.5731	0.5622	0.6146	0.6353
	15°	0.5382	0.4832	0.4444	0.3406
	30°	0.4627	0.3893	0.8311	0.8981
	45°	0.3699	0.3141	0.3203	0.7012
0.0040	0°	0.7225	0.6745	0.7419	0.9198
	15°	0.7067	0.6479	0.6955	0.8401
	30°	0.6651	0.5877	0.5952	0.6237
	45°	0.5884	0.5079	0.4628	0.6948

Undelaminated NDFF are always found to be higher than delaminated NDFF. For a particular value of laminate thickness, NDFF are found to reduce with the increase of twist angles. A typical trend is identified for NDFF with the increase of rotational speeds. At lower rotational speed (Ω =0.5), delaminated NDFF are predominantly lower than respective undelaminated NDFF, except for a few cases.

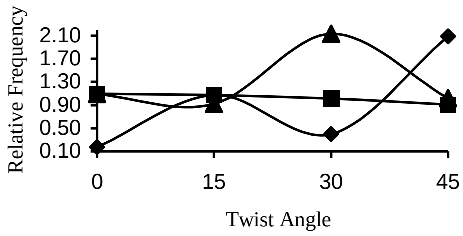
4.3 Effect of Frequency Ratio and Relative Frequency

From Table 4, frequency ratios and relative frequencies are analyzed. The trend of frequency ratio (FR) (the ratio of delaminated NDFF to undelaminated NDFF) and relative frequencies (ratio of rotating natural frequency and stationary natural frequency) at Ω =0.5, 1.0 are furnished in figures 3(a). Both delaminated and undelaminated For both lower rotating speeds (Ω=0.5), the percentage difference between maximum and minimum frequency ratio found to be 16.4%, 14.2% and 7.5% for h=0.001, 0.002 and 0.004, respectively.

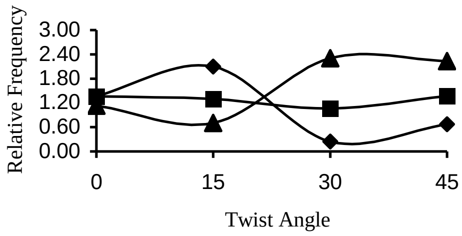
The trend of relative frequencies (RF) (the ratio of rotating non-dimensional natural frequency and stationary non-dimensional natural frequency) of bending stiff configuration at Ω=0.5, 1.0 are furnished in figures 3(b) and 3(c), respectively. At lower rotating speeds (Ω=0.5), relative frequencies of single delamination observed to be consistently higher than relative frequencies of double delamination, irrespective of twist angles. For higher rotational speeds, NDFF obtained are lower than NDFF of lower rotational speeds except a few cases. Hence it leads to fact that for higher rotating speeds, NDFF have pronounced effect. For bending stiff configuratuion, the trend of relative frequencies at Ω=0.5, 1.0 are furnished in figures 4(a) and 4(b), respectively. For both lower rotating speeds (Ω=0.5), the percentage difference between maximum and minimum relative frequencies at lower speeds found 91.8%, 56.9% and 17.2% for h=0.001, 0.002 and 0.004, respectively. On the other hand, the percentage difference between maximum and minimum relative frequencies at higher speeds obtained as 88.4%, 69.4% and 22.4% for h=0.001, 0.002, 0.004, respectively. Hence it also proved to fact that with increase of laminate thickness, relative frequencies have diminishing effect.



(a) Frequency ratios with respect to twist angles



(b) Relative frequencies at Ω=0.5 with twist angles



(c) Relative frequencies at Ω=1.0 with twist angles

h=0.001 h=0.002 h=0.004

Fig. 3 Variation of FR and RF with respect to twist angles, n=8, a/L=0.33, s/h=1000, L/s=0.7, θ_o=45°, θ_v=20°

4.4 Delamination Across Span

Effect of delamination along the span is presented in Table 5. In this analyses, relative length a/L=0.33, centered at a relative distances of d/L=0.33,0.66 from the fixed end is investigated. The delamination is considered at the mid-plane of eight layered bending stiff composite laminate. At stationary condition, NDFF are observed to decrease with the increase of twist angle. In other words, for d/L=0.33 and 0.67, NDFF found maximum value at Ψ=0° and minimum value at Ψ=45° at stationary condition. NDFF at stationary condition for untwisted conical shell are found to reduce as the delamination moves towards the free end. It is also noted that NDFF for twisted conical shells at stationary condition are identified to reduce as the delamination moves towards the free end. The centrifugal stiffening effect is predominantly identified for both d/L=0.33 and 0.66, except at Ψ=30° for d/L=0.33.

Table 5: NDFF [$\omega=\omega_n L^2 \sqrt{(\rho/E_1h^2)}$] of graphite-epoxy bending stiff conical shells with delamination along span, considering n=8, h=0.0004, a/L=0.33, s/h=1000, L/s=0.7, θ_o=45°, θ_v=20°.

Ψ	NDFF at d/L=0.33			NDFF at d/L=0.67		
	Ω=0.0	Ω=0.5	Ω=1.0	Ω=0.0	Ω=0.5	Ω=1.0
0°	0.3902	0.5223	0.6004	0.4015	0.5243	0.6041
15°	0.2339	0.4760	0.5136	0.2418	0.4310	0.4470
30°	0.1390	0.3689	0.1826	0.1450	0.1929	0.5854
45°	0.0911	0.2227	0.3786	0.0947	0.1799	0.3431

4.5 Delamination Across Thickness

Variations of NDFF with respect to relative position across thickness (h′/h) are furnished in Figure 4. It is observed that NDFF both attain a minimum value across the thickness while maximum value identified at two free surfaces. In other words, minimum values of natural frequencies are identified when delamination is located across the thickness for any values of twist angle while the two free surfaces have slightly higher values. For a particular relative position of the delamination across the thickness, it is also noted that NDFF is found to decrease with the increase in the angle of twist from 0° to 45°.

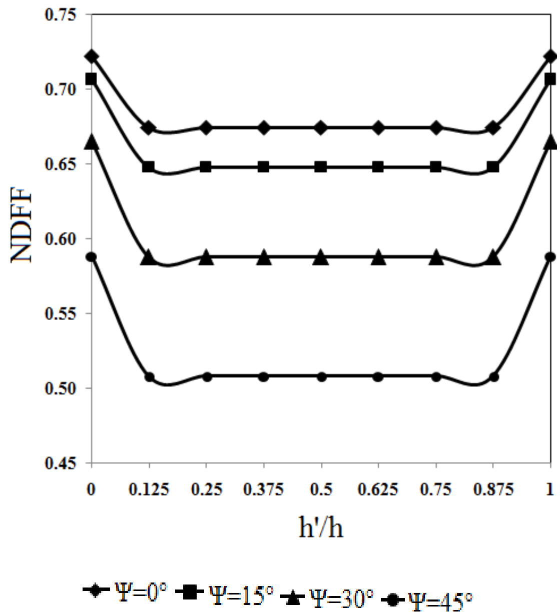


Fig 4. NDFF [$\omega=\omega_n L^2 \sqrt{(\rho/E_1 h^2)}$] of delaminated graphite-epoxy bending stiff composite conical shells for various twist angles considering n=8, h=0.004, s/h=1000, a/L=0.33, d/L=0.5, L/s=0.7, θ_o=45°, θ_v=20°.

5. MODE SHAPES

The mode shapes corresponding to NDFF are furnished in Figure 5 for various twist angles (ψ=0°, 15°, 30° and 45°), considering eight layered graphite-epoxy symmetric bending stiff composite delaminated conical shells of relative crack length a/L=0.33 centered at a relative distance d/L=0.5 from the fixed end. The fundamental frequency corresponds to the first torsional mode for all cases. It is identified that the symmetry modes are absent when twist angle is non-zero.

6. CONCLUSIONS

From the above numerical study on graphite-epoxy bending stiff composite conical shells with delamination, we can conclude that:

(1) FEM model presented in this paper validates that this mathematical formulation can be successfully employed to analyze the natural frequencies of delaminated conical shells for any particular laminate configuration.

(2) At stationary condition, NDFF are identified to reduce with the increase of twist angles.

(3) NDFF is found to decrease as the delamination moves away from the clamped end at stationary condition. The centrifugal stiffening effect is predominantly identified for both d/L=0.33 and 0.66, except at Ψ=30° for d/L=0.33.

(4) The structural stiffness increases with the increase of laminate thickness irrespective of twist angles.

(5) The mode shapes of NDFF correspond to first torsion for both twisted and untwisted cases.

(6) The natural frequencies obtained from the analyses are the first known results which can be served as reference solutions for the future investigators.

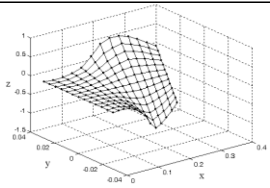
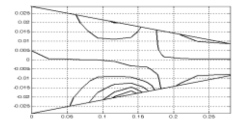
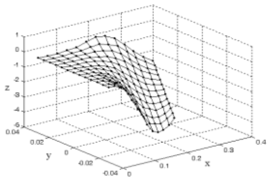
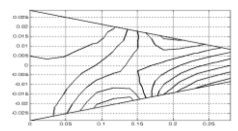
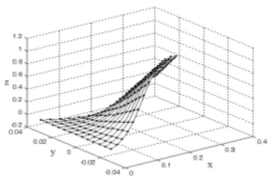
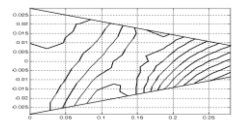
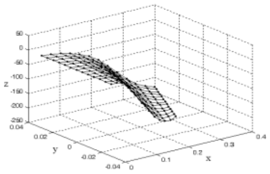
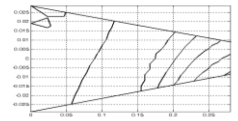
Ψ	Mode 1 (3D)	Mode 1 (2D)
0°		
15°		
30°		
45°		

Fig 5. Effect of twist on mode shapes (NDFF) of graphite-epoxy composite conical shells, n=8, Ω=0.0, h=0.004, a/L=0.33, s/h=1000, L/s=0.7, θ_o=45°, θ_v =20°.

7. ACKNOWLEDGEMENT

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8. REFERENCES

1. Bathe K. J., 1990, "Finite Element Procedures in Engineering Analysis", PHI, New Delhi.
2. Gim C.K., 1994, "Plate Finite Element Modelling of Laminated Plates", CS, 52, pp.157-168.
3. Karmakar A., Mishra T. K., Kishimoto K., 2005, "Free Vibration Characteristics of Delaminated

Composite Rotating Cantilever Shallow Shells", Proceeding of the International Conference on Fracture, March 20 – 25 (ICF 2005), Turin, Italy.
4. Karmakar A. and Sinha P. K., 2001, "Failure Analysis of Laminated Composite Pretwisted Rotating Plates", Journal of Reinforced Plastics and Composites, 20, pp.1326-1357.
5. Krawczuk M., Ostachowicz W. and Zak A., 1997, "Dynamics of Cracked Composite Material Structures", J. Computational Mechanics, 20, pp.79-83.
6. Leissa A. W., Lee J. K. and Wang A. J., 1984, "Vibrations of Twisted Rotating Blades", Journal of Vibration, Acoustics, Stress, and Reliability in Design, Trans., ASME, 106, pp.251-257.
7. Liew K. M., Lim C. M. and Ong L. S., 1994, "Vibration of pretwisted cantilever shallow conical shells", International Journal of Solids Structures, 31, pp.2463-74.
8. McGee O. G. and Chu H. R., 1994, "Three-Dimensional Vibration Analysis of Rotating Laminated Composite Blades", Journal of Engineering for Gas Turbines and Power, ASME, 116, pp.663–671.
9. Parhi P. K., Bhattacharyya S. K. and Sinha P. K., 2001, "Failure analysis of multiple delaminated due to bending and impact", Bull. Mater. Sci., 24, pp.143–149.
10. Qatu M. S. and Leissa A. W., 1991, "Natural Frequencies for Cantilevered Doubly-Curved Laminated Composite Shallow Shells", Journal of Composite Structures, 17, pp.227-255.
11. Qatu M. S. and Leissa A. W., 1991, "Vibration studies for Laminated Composite Twisted Cantilever Plates", International Journal of Mechanical Sciences, 33, pp.927-940.
12. Shen M.H.H. and Grady J. E., 1992, "Free Vibrations of Delaminated Beams", AIAA, 30, pp.1361–1370.

9. NOMENCLATURE

Symbol	Meaning
R_x	Radius of curvature in x-direction
R_y	Radius of curvature in y-direction
R_{xy}	Radius of twist
Ψ	Twist angle
Ω	Non-dimensional speed of rotation
Ω'	$= \Omega / \omega_o$
ω_o	Fundamental natural frequency of a non-rotating shell
ρ	Mass density
L	Length
a	Crack length
b_o	Reference width
ν	Poisson's ratio
h	Laminate thickness
d	Distance of centerline of delamination from clamped (fixed) end
θ_v	Vertex angle
θ_o	Base subtended angle of cone
E_1, E_2	Elastic moduli along 1 and 2 axes
G_{12}, G_{13}, G_{23}	Shear moduli along 1-2, 1-3 and 2-3 planes
[M]	Global mass matrix
[K]	Elastic stiffness matrix
$[K_\sigma]$	Geometric stiffness matrix
$\{\delta\}$	Global displacement vector
λ	Non-dimensional frequency
u_j, v_j, w_j	Nodal displacements
u'_j, v'_j, w'_j	Mid-plane displacements
θ_x, θ_y	Rotation about x and y axes
{Q}	Transverse shear resultants
[A]	Extension coefficient
[B]	Bending-extension coupling coefficient
[D]	Bending stiffness coefficients
{K}	Curvature vector
{N}	In-plane stress-resultants
{M}	Moment resultants
$\{\varepsilon\}$	Strains vector
η, ξ	Local natural coordinates of the element
L/s	Aspect ratio
n	Number of layers
ω	Natural frequency of rotating shell
NDFF	Non-dimensional fundamental natural frequency
NDSF	Non-dimensional second natural frequency
ND	No delamination or Undelaminated case

10. MAILING ADDRESS

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